Class 12 OLS Regression Basics

Dr Wei Miao

UCL School of Management

November 8, 2023

Section 1

Basics of Linear Regression

Linear Regression Models

- A simple linear regression is a model as follows.

$$Y_i = \beta_0 + x_1\beta_1 + x_2\beta_2 + ... + x_k\beta_k + \epsilon_i$$

- y_i: Outcome variable/dependent variable/regressand/response variable/LHS variable
- β: Regression coefficients/estimates/parameters; β_0: intercept
- x_k: Control variable/independent variable/regressor/explanatory variable/RHS variable
  - Lower case such as x_1 usually indicates a single variable while upper case such as X_{ik} indicates a set of several variables
- ϵ_i: Error term, which captures the deviation of Y from the prediction
  - Expected mean should be 0, i.e., $E[\epsilon|X] = 0$
  - If we take the expectation of Y, we should have

$$E[Y|X] = \beta_0 + x_1\beta_1 + x_2\beta_2 + ... + x_k\beta_k$$

Why the Name "Regression"?

- The term "regression" was coined by Francis Galton to describe a biological phenomenon: The heights of descendants of tall ancestors tend to regress down towards a normal average.
- The term "regression" was later extended by statisticians Udny Yule and Karl Pearson to a more general statistical context (Pearson, 1903).
- In supervised learning models, "regression" has a different meaning: when outcome is continuous, the task is called regression task.[1]

[1] ML models are developed by computer science; causal inference models are developed by economists.

Section 2

Estimation of Coefficients

How to Run Regression in R

- In R, there are tons of packages that can run OLS regression.
- In this module, we will be using the `fixest` package, because it's able to estimate high-dimensional fixed effects.

```
pacman::p_load(modelsummary,fixest)

OLS_result <- feols(
   fml = total_spending ~ Income, # Y ~ X
   data = data_full, # dataset from Tesco
   )
```

Report Regression Results

```r
modelsummary(OLS_result,
    stars = TRUE  # export statistical significance
  )
```

	(1)
(Intercept)	−552.235***
	(20.722)
Income	0.021***
	(0.000)
Num.Obs.	2000
R2	0.630
R2 Adj.	0.630
AIC	29 130.1
BIC	29 141.3
RMSE	351.63
Std.Errors	IID

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Parameter Estimation: Univariate Regression Case

- Let's take a **univariate regression**[2] as an example

$$y = a + bx_1 + \epsilon$$

- For each guess of a and b, we can compute the error for customer i,

$$e_i = y_i - a - bx_{1i}$$

- We can compute the **sum of squared residuals (SSR)** across all customers

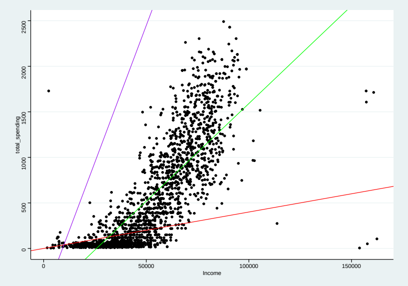
$$SSR = \sum_{i=1}^{n} (y_i - a - bx_{1i})^2$$

- **Objective of estimation**: Search for the unique set of a and b that can minimize the SSR.
- This estimation method that minimizes SSR is called **Ordinary Least Square (OLS).**

[2] Regressions with a single regressor is called univariate regressions.

Visualization: Estimation of Univariate Regression

- If in the Tesco dataset, if we regress **total spending** (Y) on **income** (X)



Model	Color	Sum of Squared Error
$Y = -552 + 0.06 * X$	Purple	1.6176047×10^{13}
$Y = 0 + 0.004 * X$	Red	5.093683×10^{11}
$Y = -552 + 0.021 * X$	Green	2.0205681×10^{9}

Multivariate Regression

- The OLS estimation also applies to multivariate regression with multiple regressors.

$$y_i = b_0 + b_1 x_1 + \ldots + b_k x_k + \epsilon_i$$

- **Objective of estimation**: Search for the **unique** set of b that can minimize the **sum of squared residuals**.

$$SSR = \sum_{i=1}^{n} (y_i - b_0 - b_1 x_1 - \ldots - b_k x_k)^2$$

Section 3

Interpretation of Coefficients

Coefficients Interpretation

- Now on your Quarto document, let's run a new regression, where the DV is $total\_spending$, and X includes $Income$ and $Kidhome$.

	(1)
(Intercept)	-316.878***
	(26.972)
Income	0.019***
	(0.000)
Kidhome	-210.613***
	(16.282)
Num.Obs.	2000
R2	0.658
R2 Adj.	0.658
AIC	28 971.2
BIC	28 988.0
RMSE	337.77
Std.Errors	IID

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

- **Controlling for** Kidhome, one unit increase in `Income` increases `totalspending` by £0.019.

Standard Errors and P-Values

- Because the regression is estimated on a random sample of the population, so if we rerun the regression on different samples, we would get a different set of regression coefficients each time.
- In theory, the regression coefficients estimates follows a **t-distribution**: the mean is the true β. The **standard error** of the estimates is the estimated standard deviation of the error.
- We can test whether the coefficients are statistically different from 0 using **hypothesis testing**.
    - Null hypothesis: the true regression coefficient β is 0
- `Income`/`Kidhome` is statistically significant at the 1% level.

R-Squared

- R-squared (R2) is a statistical measure that represents the proportion of the variance for a dependent variable that's explained by all included variables in a regression.
- Interpretation: 65.8% of the variation in `totalspending` can be explained by `Income` and `Kidhome`.
- As the number of variables increases, the R^2 will naturally increase, so sometimes we may need to penalize the number of variables using the so-called **adjusted R-squared**.

> **! Important**
>
> R-Squared is only important for supervised learning prediction tasks, because it measures the predictive power of the X. However, In causal inference tasks, R^2 does not matter much.